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The spectrum of the Ekman boundary layer problem
Borbala Gerhat (University of Bern / Queen's University Belfast)

Abstract: Originating in fluid dynamics, the study of linear stability of an Ekman boundary layer gives rise to a spectral problem for a non-selfadjoint operator matrix family. We present new eigenvalue enclosures for the point spectrum of this family and thereby solve an open problem on the existence of open sets of eigenvalues in domains of Fredholmness posed by L. Greenberg and M. Marletta in 2004. As a consequence, the spectral exactness of domain truncation approximations is valid for arbitrary Reynolds numbers.

The talk is based on a joint work with O. Ibrogimov and P. Siegl.

Metamaterial transitions on curved manifolds
Tomáš Faikl (ČVUT v Praze)

Abstract: Metamaterials possess a negative refractive index and thus present an interesting substance for designing uncommon optical effects. The talk will deal with operators encountered in an operator-theoretic description of metamaterials. Firstly, their spectra and properties will be considered in Euclidian space and later also on two-dimensional constantly-curved manifolds.

Non-self-adjoint relativistic point interactions and their approximations by non-local potentials
Lukáš Heriban (ČVUT v Praze)

Abstract: The talk deals with the Dirac operator with non–local potential given by the projection on a fixed scaled function from $L^2(\mathbb{R}) \cap L^1(\mathbb{R})$ multiply by complex matrix $\mathbb{A}$. Norm–resolvent limit of this not necessarily self–adjoint operator with non–local potential wil be discussed. Furthermore, the rigorous expression for the norm resolvent limit is compared to the formal limit of the Dirac operator with non–local potential. This formal limit corresponds to the norm–resolvent limit. In other words, contrary to the

case of scaled local potentials, renormalization of the coupling constant does not occur. This property will lead us to an extension of the definition of the Dirac operator with relativistic point interaction. Moreover, the spectrum of this newly defined operator will be discussed. A remarkable spectral transition in special cases will be presented. This spectral transition will be explained by examining ε–pseudospectrum of the operator.

The Möbius Strip
Tomáš Kalvoda (ČVUT v Praze)

Abstract: In this short talk, we will explore the history and uses of the famous Möbius strip. Last but not least, we will discuss the dynamics of a quantum particle confined to the strip.

A brief introduction to mixed volumes (miniCOURSE)
Jan Kotrbatý (Goethe University, Frankfurt)

Abstract: The notion of mixed volume arises when studying the volume of a vector sum of convex sets. In spite of having its origin in convexity, it admits interesting and deep connections to different areas of mathematics, in particular, to combinatorics and algebraic geometry.

In the first part of our talk the mixed volume will be introduced and its properties will be discussed. The second part will be devoted to more modern aspects of the theory; namely, several recent approaches to the classical results concerning mixed volumes will be presented as well as some long-term persisting open problems.

Solvable models in quasi-Hermitian quantum mechanics
David Kramár (ČVUT v Praze)

Abstract: We consider non-self-adjoint $\mathcal{PT}$-symmetric operators of Sturm-Liouville type with complex boundary conditions. We study the existence of a similarity transformation to a self-adjoint operator in dependence on the boundary parameter. We determine the values of the parameter for which the model is quasi-self-adjoint and for such values we find the self-adjoint counterpart in a closed form. In the cases when the similar self-adjoint operator does not exist we construct a generalized similarity transformation by taking the root vectors into account and find the similar operator in the closed form likewise.

Bound states in semi-Dirac semi-metals
David Krejčiřík (ČVUT v Praze)

Abstract: New insights into transport properties of nanostructures with a linear dispersion along one direction and a quadratic dispersion along another are obtained by analysing their spectral stability properties under small perturbations. The rigorous theoretical results are illustrated by numerical experiments and predictions for physical experiments are made. This is joint work with Pedro Antunes.

Effective quantum Hamiltonian in thin domains with non-homogeneity
Romana Kvasničková (ČVUT v Praze)

Abstract: This work aims to derive an effective model of the Laplacian with a non-homogeneous metric in thin domains with Neumann boundary conditions. Firstly, the Neumann Laplace operator with a non-homogeneous failure will be defined as a self-adjoint operator on the Hilbert space by an associated quadratic form. Furthermore, this work shows the convergence of this operator to the effective model in the spectral, the strong resolvent, even in the norm-resolvent sense, all of which are illustrated with a concrete example. Finally, the rate of the convergence is derived.

Szegő-type inequality for the 2D Dirac operator with infinite mass boundary conditions
Vladimir Lotoreichik (Nuclear Physics Institute, Czech Academy of Sciences)

Abstract: In this talk, we will discuss spectral features of the Dirac operator with infinite mass boundary conditions in a smooth bounded domain of $\mathbb{R}^2$. Motivated by spectral geometric inequalities, we derive a non-linear variational formulation to characterize its principal eigenvalue. This characterization turns out to be very robust and allows for a simple proof of a Szegő type inequality as well as a new reformulation of a Faber-Krahn type inequality for this operator. We will also present strong numerical evidence supporting the validity of a Faber-Krahn type inequality.

This talk is based on a joint work with Pedro Antunes, Rafael Benguria, and Thomas Ourmières-Bonafos.

Diverging eigenvalues in domain truncations of Schrödinger operators with complex potentials

Iveta Semorádová (ČVUT v Praze)

Abstract: Domain truncations of Schrödinger operators with complex potentials are known to be spectrally exact. However, several examples suggest that additional eigenvalues escaping to infinity seem to be a generic feature. We find conditions on the presence of such eigenvalues and obtain their asymptotic expansions. Our approach also yields asymptotic formulas for diverging eigenvalues in a strong coupling regime for the imaginary part of the potential.

Painting a Picture of Surreal Numbers

Rudolf Šmolka (ČVUT v Praze)

Abstract: First introduced in 1974, John H. Conway's construction of surreal numbers– the ordered field and proper class No–offers a surprisingly simple, yet substantial extension of both real and ordinal numbers. In this talk, we give a brief overview of this intriguing concept: the essential definitions, sketches of proofs and several illustrative examples, and show how the real and ordinal numbers are represented within the surreals.

Spectral bounds for 1D discrete Schrödinger operators with complex potentials

František Štampach (ČVUT v Praze)

Abstract: First, we discuss optimal spectral enclosures for discrete Laplacians on $\mathbb{Z}$ and $\mathbb{N}$ with the Dirichlet boundary condition perturbed by complex ℓ^1-potentials. Second, we present related results on a spectral stability of discrete Schrödinger operators on $\mathbb{N}$ with small complex potentials. The talk is based on joint projects with O. O. Ibrogimov, D. Krejčiřík, and A. Laptev.

Variation on harmonic theme

Lukáš Vácha (ČVUT v Praze)

Abstract: The talk will deal with the linear harmonic oscillator in the classical and especially in the quantum mechanics. Furthermore, several quantum-mechanical models with similar potentials will be investigated. In particular, spectra of the underlying

Hamiltonians will be presented and compared to the spectrum of the usual harmonic oscillator.

Clifford's Geometric Algebra in Differential Geometry

Šimon Vedl (ČVUT v Praze)

Abstract: Although W. K. Clifford introduced his geometric algebra in the nineteenth century, its potential has only been uncovered in the second half of the twentieth century. This talk is meant to give an intuitive and visual understanding of elements making up the algebra and to showcase the power of representing usual geometric operations (rotations, reflections,...) algebraically. The second half focuses on enhancing the description of embedded surfaces in Euclidean spaces using the Clifford product and then introducing the Shape operator and discussing its relation to parallel transport and curvature.