Student conference
Methods of Algebra and Functional Analysis In Applications

Děčín, Česká republika
16.–19. 5. 2022

Controlling quantum systems

Aitor Balmaseda (ČVUT v Praze)

Abstract: The development of Quantum Information Theory and the aim for building quantum computers has increased the relevance of controlling quantum systems, since being able to control states of quantum systems is a basic requirement for any quantum information processor.

The main goal of this talk is to introduce Quantum Control. We will introduce the basic ideas of Quantum Control, reviewing the standard approaches and results both in the finite-dimensional and infinite-dimensional case, including the problem of existence of dynamics for infinite-dimensional quantum systems.

Finally, we will present the Quantum Control at the Boundary scheme, a non-standard method for controlling the state of a quantum system by modifying its boundary conditions instead of relying on the action of external fields to drive the state of the system.

Discrete Schrödinger operator with a complex step potential

Vojtěch Bartoš (ČVUT v Praze)

Abstract: Consider the Hilbert space $\ell^2(\mathbb{Z})$. We study the spectrum of a class of discrete Schrödinger operators with a complex step-like potential and compare it with its continuous analogue. The potential is diagonal and dependant on a single complex parameter α, therefore, the operator is generally not self-adjoint. Finding the resolvent is formally achieved by using the Green Kernel Theorem, though the operator in question is not self-adjoint. The proposition of the theorem was proven for our special case without the self-adjointness assumption. The spectrum and its parts were identified for any α. The next step is to find spectral enclosures of the operator, which has been perturbed by an $\ell^1(\mathbb{Z})$ sequence using the Birman–Schwinger principle.

Pedal coordinates, force problems and calculus of variation

Petr Blaschke (Mathematical Institute in Opava)

Abstract: What is the best trajectory for a "gravity train" that moves through Earth using only gravity? What is the "Dark Catenary", i.e. shape of a freely hanging chain of given length which is attached to two geostationary satellites? We will make a case that these and similar questions are best to be answered in so called pedal coordinates.

Spectral asymptotics of curved metamaterials

Tomáš Faikl (ČVUT v Praze)

Abstract: We will examine spectral behaviour of an indefinite Laplacian on a rectangle embedded into a 2D Riemannian manifold of a constant curvature. This will be done using construction of singular sequences on this rectangle.

Quantum Hall effect

Marie Fialová (TBA)

Abstract: This talk intends to introduce terms that emerge when studying the (Fractional) Quantum Hall effect.

Pseudo numerical ranges

Borbala Gerhat (ČVUT v Praze)

Abstract: The numerical range of a linear operator in a Hilbert space is a powerful tool in its a priori spectral analysis. We recall properties of this classical object and its generalisation to families of linear operators, as well as the quadratic numerical range for (families of) linear operator matrices. In order to overcome some limitations of these objects, we introduce the pseudo (quadratic) numerical range. We explain the ideas leading to these new notions and show examples to illustrate their robust properties with respect to spectral enclosures.

Based on joint work with Christiane Tretter, University of Bern, Switzerland.

Non-relativistic limit of the Dirac operator

Lukáš Heriban (ČVUT v Praze)

Abstract: The physically interesting principal of the non-relativistic limit is studied for almost all variations of the Dirac operator. We investigate the non–relativistic limit of our relativistic model mainly for two reasons. Firstly, we would like to have our Schrödinger theory contained in the relativistic theory, i.e., it is reasonable to have the non-relativistic model as the limiting case of the relativistic one. Secondly, in some cases it is useful to use non-relativistic theory with some relativistic correction instead of the much harder Dirac theory. In this talk, we will go trough the classical proof of the non–relativistic limit of the free Dirac operator presented in [3]. To achieve such limit the Dirac operator must be adjust correctly because of few physical reasons, which will be explained. Then by letting c, the speed of light, tend to infinity we will get the sought limit. Eventually, we will use this classical result to proof the non-relativistic limit of the operator of the relativistic point interaction and of the Dirac operator perturbed by the regular non-local potential.

References:

[1] S. Benvegnu, L. Dabrowski, Relativistic point interaction in one dimension. Letters in Mathematical Physics 30, 1994, 159-167.

[2] F. Gesztesy, H. Grosse, B. Thaller, A rigorous approach to relativistic corrections of bound state energies for spin-1/2 particles. Annales de l'I.H.P. Physique théorique 40, 1984, 159-174.

[3] B. Thaller, The Dirac equation. Springer-Verlag, Berlin Heidelberg, 1992.

Spectral enclosures for discrete bilaplacian with complex potential

Tomáš Hrdina (ČVUT v Praze)

Abstract: I study resolvent and spectrum of the dicsrete bilaplacian on $\mathbb{Z}$ and the changes of spectrum caused by a compact diagonal perturbation. The potential is generated by ℓ^1 sequence v. My aim is to find an area which includes the whole spectrum of perturbated operator, since we can not find it exactly for general potential. Having used the Birman-Schwinger principle I obtained non-optimal spectral enclosures and a hypotesis for optimal boundary. The boundary curves are dependent only on the ℓ^1-norm of the sequence v.

WKBJ analysis and reaction-diffusion equations

Juraj Kováč (ČVUT v Praze)

Abstract: Asymptotic analysis has yielded a number of worthwhile insights in multiple fields, including pattern formation. With this in mind, we present a set of approximation theorems that generalize scalar WKBJ (more precisely, Liouville-Green) theory to multicomponent systems, relying on their spectral properties. Subsequently, we relate these properties to a typical reaction-diffusion (Turing) system and demonstrate the main results obtained via a WKBJ analysis

Stability of spectra for 1-parametric Dirac operators on a half-line

David Kramár (ČVUT v Praze)

Abstract: We consider 1-parametric Dirac operators on a half-line with infinite-mass boundary conditions perturbed by (generally) non-self-adjoint L^1 potentials. For a given matrix-valued potential we derive a sufficient explicit condition for the stability of the spectrum of the perturbed operator.

Absence of embedded boundstates in semi-Dirac semi-metals

Romana Kvasničková (ČVUT v Praze)

Abstract: Semi-Dirac semi-metals are materials, which behave like both conventional zero-gap semiconductors and materials of graphene type in a sense. First, we will introduce the low-energy description of the unperturbed system by the suitable definition of the corresponding Hamiltonian. Then the virial theorem will be proved and the conditions of absence of eigenvalues embedded in the essential spectrum will be formulated.

Self-adjointness for the MIT bag model on a cone

Vladimir Lotoreichik (Nuclear Physics Institute, Czech Academy of Sciences)

Abstract: We will discuss the massless Dirac operator D on an unbounded three-dimensional circular cone. We define this operator on four-component H^1-functions satisfying the MIT bag boundary conditions on the boundary of the cone. The closed Dirac operator D is symmetric and the natural question arises whether it is self-adjoint or it has non-trivial deficiency indices. We prove that D is self-adjoint for convex cones and provide a numerical evidence for its self-adjointness also for non-convex cones. The situation is different from the Dirichlet Laplacian on the cone (defined on H^2-functions with vanishing trace), where there is a transition between self-adjointness and deficiency indices $(1,1)$ for the opening angle of the cone exceeding the critical value. As a by-product of our analysis we obtain a Hardy inequality for D. These results are obtained in collaboration with Biagio Cassano.

Approximations of relativistic point and delta-shell interactions by regular potentials

Matěj Tušek (ČVUT v Praze)

Abstract: We start with a brief exposition of how the one-dimensional Dirac operator with a general point interaction can be approximated by the Dirac operator with either local or non-local regular potential. Then we introduce the two-dimensional Dirac operator with a quite general singular interaction supported on a closed curve and find approximating sequences of operators with regular potentials also in this setting. Finally, a surprising effect of a renormalization of the coupling constant will be briefly discussed and related to the Klein paradox-for a complete explanation, don't miss the talk of Lukáš Vácha!

The Klein paradox

Lukáš Vácha (ČVUT v Praze)

Abstract: The talk deals with scattering problem for the Dirac equation. In particular, reflection and transmission coefficient for the step-like, rectangle, and delta potentials are calculated. An unexpected dependence of these coefficients on energies, known as the Klein paradox, is discussed. Finally, we compare the scattering coefficients for the delta interaction and its approximation by scaled rectangular potentials.

Reverse isoperimetric inequality for the lowest Robin eigenvalue of a triangle

Vu Thi Bich Tuyen (ČVUT v Praze)

Abstract: We consider the Laplace operator on a triangle, subject to attractive Robin boundary conditions. We prove that the equilateral triangle is a local maximiser of the lowest eigenvalue among all triangles of a given area provided that the negative boundary parameter is sufficiently small in absolute value, with the smallness depending on the area only. Moreover, using various trial functions, we obtain sufficient conditions for the global optimality of the equilateral triangle under fixed area constraint in the regimes of small and large couplings. We also discuss the constraint of fixed perimeter. These are joint works with David Krejčiřík and V. Lotoreichik