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In **A**pplications

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Pseudospectrum and spectral stability of the discrete Schrödinger operator with a complex step potential

Vojtěch Bartoš (ČVUT v Praze)

Abstract: We study spectral properties of a Schrödinger operator H_α on $\ell^2(\mathbb{Z})$ with a step-like potential and complex coupling constant α. The first of said properties are the pseudospectra. We obtained estimates of the resolvent operator's norm from above and below. These estimates give us two sets, one superset and one subset of the pseudospectrum. We have also derived asymptotic formulas for pseudospectra. Utilizing the Birman-Schwinger principle, we study the existence of weak-coupled eigenvalues as the second property. First, we consider the Laplace operator (i.e., we set $\alpha = 0$) and provide an analogue to the continuous case, showing that under certain restrictions on the potential V, a unique eigenvalue exists. An analogous result is obtained in the discrete setting. Next, we consider the parameter α with a non-zero imaginary part. We show that given some restrictions on the potential V, we can rule out the existence of eigenvalues; in other words, H_α exhibits spectral stability.

Calculus using hypergeometric functions

Petr Blaschke (Mathematical Institute in Opava)

Abstract: How to get better at integration, differentiation, solving differential equation, discrete summation and other of the myriad tasks that fills a life of a modern mathematical analyst? Simple! Just use hypergeometric functions!

Criticality analysis of fractional discrete Laplacians

Borbala Gerhat (ČVUT v Praze)

Abstract: We study the existence of non-trivial lower bounds for powers of the discrete Dirichlet Laplacian on the half-line. Unlike in the continuous setting, where both $-\Delta$ and $(-\Delta)^2$ admit a non-trivial Hardy type inequality, their discrete analogues exhibit a different behaviour. While the discrete Laplacian is subcritical, its square is critical. The threshold where the criticality of the powers $(-\Delta)^\alpha$ appears turns out to be $\alpha = 3/2$. We provide corresponding (non-optimal) Hardy type inequalities in the subcritical regime.

Based on joint work with D. Krejčiřík and F. Štampach.

Pseudospectra in an operator-theoretic description of black holes

Jan Havel (ČVUT v Praze)

Abstract: We are in the era of gravitational wave astronomy and there is an ongoing research in testing the nature of black holes with gravitational wave observations. The problem of obtaining the frequencies which encode the geometric information about black holes can be cast as a spectral problem for a non- self-adjoint operator which is obtained by rewriting a wave equation as a matrix evolution problem. Using the Lax-Milgram theorem, we properly define a simplified version of the operator introduced in [1] for the Schwarzschild black hole case. Moreover, we show how its components can be used to find its spectrum. We also discuss pseudospectra and their use in black hole spectroscopy.

References:

[1] J. L. Jaramillo, R. Panosso Macedo, and L. Al Sheikh, Pseudospectrum and Black Hole Quasi-normal Mode Instability, Phys. Rev. X 11, 031003 (2021).

Non-local delta shell interactions for the Dirac operator

Lukáš Heriban (ČVUT v Praze)

Abstract: The recent development of new techniques of boundary triples allowed us to study self-adjointness of the Dirac operator perturbed by the singular potential $D_l^{\mathbb{A}} = D + \mathbb{A}\delta_\Sigma$, where $\mathbb{A}$ is arbitrary hermitian matrix and δ_Σ stands for the single-layer distribution supported on closed non-self-intersecting C^2 curve or surface Σ in $\mathbb{R}^n$, $n \in \{2, 3\}$; cf. [1,2]. It was proved that the operator $D_l^{\mathbb{A}}$ is actually a self-adjoint extension of a symmetric operator $D_0\varphi = D\varphi$, $\varphi \in H_0^1(\mathbb{R}^n \setminus \Sigma)$. Contrary to the general belief, by studying seemingly similar formal operator $D_{nl}^{\mathbb{A}} = D + \mathbb{A}|\delta_\Sigma\rangle\langle\delta_\Sigma|$ we will be able to define completely new self-adjoint extensions of the operator D_0. Let us mention that this is not the case in one dimension, where operators $D_l^{\mathbb{A}}$ and $D_{nl}^{\mathbb{A}}$ are one and the same. To promote the interest of our results we will construct regular approximations of the operator $D_{nl}^{\mathbb{A}}$ and we will prove the norm-resolvent convergence for such approximations.

References:

[1] J. Behrndt, M. Holzmann, T. Ourmières-Bonafas, K. Pankrashkin, Two-dimensional Dirac operators with singular interactions supported on closed curves. Journal of Functional Analysis, vol. 279 (2020).

[2] J. Behrndt, M. Holzmann, C. Stelzer, G. Stenzel, Boundary triples and Weyl functions for Dirac operators with singular interactions. arXiv:2211.05191 (2022).

Spectrum of the discrete bilaplace operator with complex potential

Tomáš Hrdina (ČVUT v Praze)

Abstract: We study the spectrum of the discrete bilaplace operator on $\ell^2(\mathbb{Z})$ perturbed by a complex potential generated by a complex sequence $v \in \ell(\mathbb{Z})$. We have already found certain spectral enclosures using the Birman-Schwinger principle and have a conjecture on optimal spectral enclosures. The talk will deal with a proof of the conjecture. It is based on an optimal estimate of the norm of the Birman-Schwinger operator. The conjecture was based on some numerical calculations, but the proof is purely analytic and is based on basic principles of complex analysis. Moreover, we will discuss the possibility of proving the absence of eigenvalues of the perturbed operator in the interval $(0, 16)$, which is a subset of the essential spectrum of the unperturbed bilaplace operator.

Visualising Quantum Mechanics

Michal Jex (ČVUT v Praze)

Abstract: Quantum mechanics can be built from the ground up as a rigorous mathematical theory. As such one is able to derive precise statements and draw conclusion. However, the interpretation and physical intuition might be lost or obscured within the claims. As the well-known saying goes "A picture is worth a thousand words" we present visually some basic and some more interesting examples from the field of quantum mechanics and their underlying physical phenomena. The lecture is based on [1,2].

References:

[1] Thaller, Bernd, Visual quantum mechanics: selected topics with computer-generated animations of quantum-mechanical phenomena. Springer Science & Business Media (2001).

[2] Thaller, Bernd, Advanced visual quantum mechanics. Springer Science & Business Media (2005).

Kato's Upper and Lower Estimates of Eigenvalues

Tomáš Kalvoda (ČVUT v Praze)

Abstract: In this short presentation, I will review an interesting old result by Tosio Kato (1949) concerning both upper and lower estimates of eigenvalues of a self-adjoint operator in Hilbert space. The proof of his result is quite simple and relies only on the Spectral Theorem. The talk is designed to be accessible to students and at the same time potentially interesting to specialists in the field.

Discrete Dirac operator and its non-relativistic limit

Ruben Karapetyan (ČVUT v Praze)

Abstract: In this talk, the main goal is to prove that the discrete Dirac operator, shifted by mc^2, converges to the discrete Laplace operator in the norm resolvent sense as c approaches infinity. Moreover, we obtain the full Taylor expansion of the discrete Dirac operator's resolvent in the neighborhood of $c = +\infty$. Lastly, we investigate how adding a bounded potential influences the convergence of the Discrete Dirac operator in the norm resolvent sense.

Spectral stability of a relativistic quantum particle on a half-line

David Kramár (ČVUT v Praze)

Abstract: We consider a one-parametric self-adjoint Dirac operator $\mathcal{D}_\alpha$ on a half-line perturbed by a multiplication operator generated by an L^1-matrix-valued function. We derive a sufficient condition on the stability of the spectrum of the perturbed operator and discuss the optimality of the obtained results.

Weakly coupled bound states in semi-Dirac semi-metals

Romana Kvasničková (ČVUT v Praze)

Abstract: Semi-Dirac semi-metals are recently constructed nanostructures, which behave like both conventional zero-gap semiconductors and materials of graphene type in a sense. The main objective of this talk is to formulate sufficient conditions of existence and derive asymptotics of weakly coupled eigenvalues, i.e. in the asymptotic regime with the perturbation εV and $\varepsilon \to 0$ when the essential spectrum is preserved, on the strip oriented in the direction of the x- or y-axis. The method of the proof is based on the Birman-Schwinger principle and follows the idea of [1], where J.-C. Cuenin and P. Siegl analyze the weak coupling limit for the one-dimensional Dirac operator.

References:

[1] J.-C. Cuenin, P. Siegl, Eigenvalues of one-dimensional non-self-adjoint Dirac operators and applications, Letters in Mathematical Physics 108, (2018).

Dirac operator with a discontinuous complex potential

Duc Tho Nguyen (ČVUT v Praze)

Abstract: In this presentation, we will talk about spectral properties (spectrum and resolvent) of the non-selfadjoint Dirac operator, for $m \geq 0$,

$$\mathscr{L}_m = \begin{pmatrix} m & -\partial_x \\ \partial_x & -m \end{pmatrix} + i\,\mathrm{sgn}(x)\,\mathrm{I}_2, \qquad \mathrm{I}_2 := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Then we will discuss about its point interaction:

$$\mathscr{L}_{\alpha,m} = \mathscr{L}_m + \alpha\delta_0\mathrm{I}_2, \qquad \alpha \in \mathbb{C},$$

here, δ_0 is the Dirac distribution supported at $x = 0$.

Groups Like Stars in the Night Sky

Rudolf Šmolka (ČVUT v Praze)

Abstract: A group is a set with three operations: multiplication, inversion and the unit element, which can be thought of as maps from the cartesian product of the group with itself, the group itself and the one-point set, respectively. The relations between these maps can be described in the form of three commutative diagrams. But commutative diagrams make sense in any category, which leads to the definition of a group object. In this talk we will see what we can learn about these generalizations of a group - first in functor categories and then, through the use of the Yoneda lemma - in any (locally small) category with finite products and a terminal object.

New families of orthogonal polynomials generated from the level 1 solution of the Heun equation and spectra of the corresponding Jacobi matrices

Patrik Šnauko (ČVUT v Praze)

Abstract: A correspondence between the so-called Heun function and orthogonal polynomials was discovered at the beginning of this century by G. Valent. In this talk, we will focus on the level 1 solutions of the Heun equation. We introduce some symmetries which allow generating new solutions from already known ones. Finally, we pick a family and introduce results obtained from the Darboux method about measure of orthogonality of the corresponding orthogonal polynomials sequence. Knowledge of the measure of orthogonality for orthogonal polynomials yields knowledge of spectrum of the corresponding Jacobi matrix since its spectrum supports the measure.

Magnetic effects in the spectrum of laterally coupled layers

Ondřej Šrámek (ČVUT v Praze)

Abstract: In these speech, we will introduce classical models with mangetic laplacian motivated by Quantum Hall efect, such as Iwatsuka model. We will introduce terms of absolute continuous spectrum and direct integral. The last point we will discuss is a system of laterally coupled layers with Neumann window.

The Schur test

František Štampach (ČVUT v Praze)

Abstract: We discuss the classical test of Issai Schur (1911) in its generalized form. As an application, we compute the operator norm of the Hilbert matrix.

Superbrief introduction to supersymmetric methods

Matěj Tušek (ČVUT v Praze)

Abstract: In this supershort series of lectures I will try to describe the foundations of supersymmetric quantum mechanics. Supersymetric Dirac operators will serve us as superimportant examples. Beware that the prefix "super" will be superoverused during the talks to subconsciously impress the audience.

Shape sensitivity analysis of a Maxwell's cavity problem

Michele Zaccaron (ČVUT v Praze)

Abstract: Starting from time-harmonic Maxwell's equations in a bounded domain Ω of $\mathbb{R}^3$, representing a perfect conductor, we obtain a second order differential system of curl-curl type. The topic of the talk will focus on the behaviour of the eigenvalues of the system as the shape of the domain Ω varies. We will discuss the eigenproblem and the properties of its spectrum, introducing useful tools as well as presenting some recent results.