MAFIA - the seminar you can't refuse

Schur complement dominant operator matrices

Borbala Gerhat

University of Bern / Queen's University Belfast

12 December (Thursday!) 2019
13:30–14:30
in T112

Fakulta jaderná a fyzikálně inženýrská, ČVUT v Praze
Trojanova 13, 12000 Praha

Abstract: In mathematical physics, matrix (differential) operators arise naturally in applications as coupled systems of partial differential equations. Up to now, the spectral analysis of such problems has typically been tackled by means of perturbation theory. We propose to view operator matrices in a more general setting, which allows our results to fully abstain from any perturbative argument. Rather than requiring the matrix to act in a Hilbert space $\mathcal{H}$, we extend its action to a suitable distributional triple $\mathcal{D} \subset \mathcal{H} \subset \mathcal{D}'$ and restrict it to its maximal domain in $\mathcal{H}$. The crucial point in our approach is the choice of the spaces $\mathcal{D}$ and $\mathcal{D}'$ which are essentially determined by the Schur complement of the matrix. We show spectral equivalence between the resulting operator matrix in $\mathcal{H}$ and its Schur complement, eventually implying closedness and non-empty resolvent set of the matrix. Finally, we apply our abstract results to the damped wave equation with possibly unbounded and singular damping, as well as to more general second order matrix differential operators with singular coefficients. By means of our methods, the previously used regularity assumptions can be lowered substantially in both cases.