Abstract: We consider the linear damped wave equation on possibly unbounded domains with the damping being allowed to become unbounded at infinity and we investigate newly arising phenomena. Based on a careful analysis of the associated quadratic operator function, having the form of a Schrödinger operator with a complex potential, we prove the generation of a contraction semigroup. Besides showing the convergence of the non-real eigenvalues in the asymptotic regime of an infinite damping on a subdomain, we establish the presence of the negative essential spectrum touching zero and spectral instabilities (pseudospectrum). The occurrence of the latter two features turns out to be a robust effect that cannot be easily canceled by adding a positive potential and so the exponential estimate of the semigroup is often lost and the spectrum is very sensitive to perturbations.