Abstract: Representation theorems are useful tools in characterising and dealing with the mathematical objects they are related with. One famous example is the spectral theorem for (unbounded) self-adjoint operators, that allows to work with operators on a Hilbert space as if they were multiplication operators on a space of square integrable functions.

We will consider the case of Hermitian quadratic forms, which are bilinear functions on Hilbert spaces and that constitute another example of maps on a Hilbert space that admit a representation theorem. These representation theorems usually come with an important drawback, the quadratic form needs to be semibounded. There are many important examples where this condition is not met and therefore alternative representation theorems are needed.

We will provide a representation theorem for non-semibounded Hermitian quadratic forms based on a decomposition in terms of a direct integral of the Hilbert space and the notion of orthogonal additivity. We will introduce the main ideas in a constructive way and provide meaningful examples.