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Schur complement dominance with applications to wave equations and Dirac operators

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Abstract: Due to the matrix structure, spectral problems arising from systems of coupled linear partial differential equations can be intrinsically challenging. A successful approach motivated already on the level of scalar matrices is to relate the operator matrix (to be implemented in a product Hilbert space) to one of its Schur complements. In the unbounded operator setting, we introduce a robust abstract method to rigorously establish this connection and thereby extend previous approaches which use relative boundedness within the matrix entries. The cornerstones of our method are on one hand a Schur complement which is dominant in a suitable sense with respect to the entries. On the other hand, we employ a distributional approach and implement the matrix operations in larger spaces before restricting to the maximal domain in the underlying product Hilbert space. This allows us to essentially pass from a well-behaved representation of the Schur complement (for instance by its sesquilinear form) to a representation of the operator matrix and show that the latter is densely defined and has non-empty resolvent set on the resulting maximal domain, as well as to relate the (point and essential) spectra of matrix and Schur complement.

We illustrate this abstract framework and present a semigroup generation result for a wave equation with accretive damping in a weighted space. The dominant Schur complement therein is implemented as the form representation of a Schrödinger-type operator with accretive potential in a weighted space. Moreover, we use Schur complement dominance to establish the self-adjointness of Dirac operators with Coulomb-like potentials which satisfy a Hardy-Dirac inequality. In both problems, the method allows for a generalisation (to in some sense naturally minimal assumptions) with respect to previously existing results.