MAFIA seminář

On the similarity of Sturm-Liouville operators with non-Hermitian boundary conditions to self-adjoint and normal operators

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Abstract: We consider one-dimensional Schrödinger-type operators in a bounded interval with non-self-adjoint Robin-type boundary conditions. It is well known that such operators are generically conjugate to normal operators via a similarity transformation. Motivated by recent interests in quasi-Hermitian Hamiltonians in quantum mechanics, we study properties of the transformations in detail. We show that they can be expressed as the sum of the identity and an integral Hilbert-Schmidt operator. In the case of parity and time reversal boundary conditions, we establish closed integral-type formulae for the similarity transformations, derive the similar self-adjoint operator and also find the associated "charge conjugation" operator, which plays the role of fundamental symmetry in a Krein space reformulation of the problem.

Conceptually, the described ideas will be explained in details on an exactly solvable example where all relevant objects can be found in closed formulae forms. Then general claims will be formulated and comments on ideas of the proofs will be given. The talk should be accessible to students with basic knowledge on operators in Hilbert spaces (e. g. FA1 course).

The talk is partially based on the joint work with D. Krejčiřík (NPI ASCR, Řež) and J. Železný (FZU ASCR, Prague):

D. Krejčiřík, P. Siegl and J. Železný: On the similarity of Sturm-Liouville operators with non-Hermitian boundary conditions to self-adjoint and normal operators, preprint available at arXiv:1108.4946