



MAFIA - the seminar you can't refuse

Quantum Systems at the Brink

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12:00–13:00

in T212

Fakulta jaderná a fyzikálně inženýrská ČVUT
Trojanova 13, 12000 Praha

Abstract: (joint work with with Dirk Hundertmark and Markus Lange)

One of the crucial properties of a quantum system is the existence of bound states. The existence of eigenvalues below the essential spectrum is well understood. They exhibit exponential decay, and their existence is linked to the energy gap. However, the situation at the threshold is much more subtle. There are two challenging problems for the states at the threshold—their existence and asymptotic behaviour. Since the usual methods for addressing these problems need a safety distance to the essential spectrum, they cannot be applied in critical cases, when an eigenvalue enters the continuum.

We present necessary and sufficient conditions for Schrödinger operators to have a zero-energy bound state. Our sharp criteria show that the existence and non-existence of zero-energy ground states depends strongly on the dimension and the asymptotic behaviour of the potential. There is a spectral phase transition with dimension four being critical.

Furthermore, we present a recently developed method to address the decay rate behaviour. As an illustration of the application, we derive sharp upper and lower bounds for the asymptotic behaviour of the ground state of critical helium type systems at the threshold of the essential spectrum. This is the first proof of the precise asymptotic behaviour of the ground state for this benchmark problem in quantum chemistry.