MAFIA - the seminar you can’t refuse

Derivations of Leavitt Path Algebras

Viktor Lopatkin
ČVUT v Praze

19 February 2019
13:15–14:15
in T112

Fakulta jaderná a fyzikálně inženýrská, ČVUT v Praze
Trojanova 13, 12000 Praha

Abstract: Given a row-finite directed graph $\Gamma$ and a field $K$, G. Abrams and A. Pino in [2], and independently P. Ara, M.A. Moreno, E. Pardo in [5], introduced the Leavitt path algebra $L_K(\Gamma)$. This algebra is an algebraic analog of graph Cuntz–Krieger $C^*$-algebras. These algebras have attracted significant interest and attention, not only from ring theorists, but from analysts working in $C^*$-algebras, group theorists, and symbolic dynamicists as well [3, 4, 6, 1].

The connections between Leavitt path algebra and $C^*$-algebras can be described as below. Let $\Gamma$ be a graph. Very roughly, the process by which a $C^*$-algebra is associated to $\Gamma$ consists of decorating the vertices with orthogonal projections on a Hilbert space $H$ and the edges, with suitable operators. The ensuing $C^*$-subalgebra of the bounded linear operators $B(H)$ is then the graph $C^*$-algebra $C^*(\Gamma)$. The Leavitt path algebras, denoted as $L(\Gamma)$, are the algebraic siblings of the aforementioned graph $C^*$-algebras and are constructed over an arbitrary field (whereas here $C^*$-algebras will always be over the complex numbers). Both classes of algebras, $L(\Gamma)$ and $C^*(\Gamma)$, share a beautiful interplay between highly visual properties of the graph and algebraic/analytical properties of the corresponding underlying graphs.
In this talk, we describe the $K$-module $HH^1(L_K(\Gamma))$ (=the first Hochschild cohomology) of outer derivations of the Leavitt path algebra $L_K(\Gamma)$ of a row-finite graph $\Gamma$ with coefficients in an associative commutative ring $K$ with unit. We explicitly describe a set of generators of $HH^1(L_K(\Gamma))$ and relations among them. We also describe a Lie algebra structure of outer derivation algebra of the Toeplitz algebra. We prove that every derivation of a Leavitt path algebra can be extended to a derivation of the corresponding $C^*$-algebra.

REFERENCES