MAFIA - the seminar you can't refuse

Derivations of Leavitt Path Algebras

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ČVUT v Praze

19 February 2019
13:15–14:15
in T112

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Trojanova 13, 12000 Praha

Abstract: Given a row-finite directed graph Γ and a field K, G. Abrams and A. Pino in [2], and independently P. Ara, M.A. Moreno, E. Pardo in [5], introduced the *Leavitt path algebra* $L_K(\Gamma)$. This algebra is an algebraic analog of graph Cuntz–Krieger C^*-algebras. These algebras have attracted significant interest and attention, not only from ring theorists, but from analysts working in C^*-algebras, group theorists, and symbolic dynamicists as well [3, 4, 6, 1].

The connections between Leavitt path algebra and C^*-algebras can be described as below. Let Γ be a graph. Very roughly, the process by which a C^*-algebra is associated to Γ consists of decorating the vertices with orthogonal projections on a Hilbert space $\mathcal{H}$ and the edges, with suitable operators. The ensuing C^*-subalgebra of the bounded linear operators $B(\mathcal{H})$ is then the graph C^*-algebra $C^*(\Gamma)$. The Leavitt path algebras, denoted as $L(\Gamma)$, are the algebraic siblings of the aforementioned graph C^*-algebras and are constructed over an arbitrary field (whereas here C^*-algebras will always be over the complex numbers). Both classes of algebras, $L(\Gamma)$ and $C^*(\Gamma)$, share a beautiful interplay between highly visual properties of the graph and algebraic/analytical properties of the corresponding underlying graphs.

In this talk, we describe the K-module $HH^1(L_K(\Gamma))$ (=the first Hochschild cohomology) of outer derivations of the Leavitt path algebra $L_K(\Gamma)$ of a row-finite graph Γ with coefficients in an associative commutative ring K with unit. We explicitly describe a set of generators of $HH^1(L_K(\Gamma))$ and relations among them. We also describe a Lie algebra structure of outer derivation algebra of the Toeplitz algebra. We prove that every derivation of a Leavitt path algebra can be extended to a derivation of the corresponding C^*-algebra.

References

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