**Abstract:** We present recent results obtained in collaboration with Dario Prandi, about Hardy inequalities in the $n$-th Heisenberg group, and subject of the preprint [1]. In particular, we show that, contrary to the Euclidean case, a radial Hardy inequality, i.e., a Hardy inequality taking into account only the directional derivative w.r.t. the sub-Riemannian distance, does not hold in this context for any dimension. Motivated by this fact, we then suggest the study of a non-radial Hardy inequality, based on the construction of specific polar-type coordinates following from the explicit synthesis of sub-Riemannian geodesics. We prove a sharp weighted non-radial inequality that imply (non-sharp) bounds for the non-radial Hardy constant on homogeneous cones. We underly through the latter a strong difference with respect to the Euclidean case.

**REFERENCES**