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Strong coupling asymptotics for $\delta$-interactions supported by curves with cusps

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Abstract: Let $\Gamma \subset \mathbb{R}^2$ be a simple closed curve which is smooth except at the origin, at which it has a power cusp and coincides with the curve $|x_2| = x_1^p$ for some $p > 1$. We study the eigenvalues of the Schrödinger operator $H_\alpha$ with the attractive $\delta$-potential of strength $\alpha > 0$ supported by $\Gamma$, which is defined by its quadratic form

$$H^1(\mathbb{R}^2) \ni u \mapsto \int_{\mathbb{R}^2} |\nabla u|^2 \, dx - \alpha \int_{\Gamma} u^2 \, ds,$$

where $ds$ stands for the one-dimensional Hausdorff measure on $\Gamma$. It is shown that if $n \in \mathbb{N}$ is fixed and $\alpha$ is large, then the well-defined $n$th eigenvalue $E_n(H_\alpha)$ of $H_\alpha$ behaves as

$$E_n(H_\alpha) = -\alpha^2 + 2 \frac{2}{p+2} E_n \alpha^{\frac{2}{p+2}} + O(\alpha^{\frac{6}{p+2} - \eta}),$$

where the constants $E_n > 0$ are the eigenvalues of an explicitly given one-dimensional Schrödinger operator determined by the cusp, and $\eta > 0$. Both main and secondary terms in this asymptotic expansion are different from what was observed previously for the cases when $\Gamma$ is smooth or piecewise smooth with non-zero angles.