MAFIA - the seminar you can't refuse

Strong coupling asymptotics for δ-interactions supported by curves with cusps

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Abstract: Let $\Gamma \subset \mathbb{R}^2$ be a simple closed curve which is smooth except at the origin, at which it has a power cusp and coincides with the curve $|x_2| = x_1^p$ for some $p > 1$. We study the eigenvalues of the Schrödinger operator H_α with the attractive δ-potential of strength $\alpha > 0$ supported by Γ, which is defined by its quadratic form

$$H^1(\mathbb{R}^2) \ni u \mapsto \iint_{\mathbb{R}^2} |\nabla u|^2 \, \mathrm{d}x - \alpha \int_\Gamma u^2 \, \mathrm{d}s,$$

where $\mathrm{d}s$ stands for the one-dimensional Hausdorff measure on Γ. It is shown that if $n \in \mathbb{N}$ is fixed and α is large, then the well-defined nth eigenvalue $E_n(H_\alpha)$ of H_α behaves as

$$E_n(H_\alpha) = -\alpha^2 + 2^{\frac{2}{p+2}} \mathcal{E}_n \, \alpha^{\frac{6}{p+2}} + \mathcal{O}(\alpha^{\frac{6}{p+2} - \eta}),$$

where the constants $\mathcal{E}_n > 0$ are the eigenvalues of an explicitly given one-dimensional Schrödinger operator determined by the cusp, and $\eta > 0$. Both main and secondary terms in this asymptotic expansion are different from what was observed previously for the cases when Γ is smooth or piecewise smooth with non-zero angles.