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Infinitely many solutions to generalized quasilinear critical Schrödinger equations in $\mathbb{R}^N$

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Abstract: In this talk, we will discuss some recent results concerning quasilinear Schrödinger problems in the entire $\mathbb{R}^N$ of the type

$$(1) \qquad -\Delta_p u - \frac{\alpha}{2}\Delta_p(|u|^\alpha)|u|^{\alpha-2}u = \lambda V(x)|u|^{k-2}u + \beta K(x)|u|^{p_\alpha^*-2}u \quad \text{in } \mathbb{R}^N,$$

with $1 < p < N$, $\alpha > 0$, $\max\{1, \alpha\} < k < p_\alpha^* := \max\{p^*, \alpha p^*\}$, V, K nonnegative nontrivial weights and λ, β positive real parameters associated to the subcritical and critical terms, respectively. Quasilinear equations of the type (1) are related to models of several physical phenomena such as plasma physics, high-power ultrashort laser in matter, fluid mechanics, the theory of Heisenberg ferromagnets and magnons.

Our analysis is twofold since the two cases $\alpha > 1$ and $0 < \alpha < 1$ give rise to a different nature of the equation under consideration, indeed, in the first case the term $\Delta_p(|u|^\alpha)|u|^{\alpha-2}u$ is degenerate at $u = 0$, while in the latter case it becomes singular when $u = 0$. For this reason, we will justify the definition of the critical exponent p_α^* in terms of compactness of a suitable embedding and of nonexistence of solutions beyond p_α^*. Moreover, the well definition of the functional forces to

consider the range of the exponent k enlarged on one side and restricted on the other side according to the value of α.

In particular, under suitable conditions on the subcritical exponent k, we obtain multiplicity results with negative and positive energy depending on the range of the parameters λ, β. We analyze also the case of nonnegative nontrivial weights satisfying some symmetry conditions with respect to a certain group $T \subset O(N)$, where $O(N)$ is the group of orthogonal linear transformations in $\mathbb{R}^N$.

Our proofs rely on variational tools, including concentration compactness principles needed for overcoming the double loss of compactness due both to the critical exponent and to the unboundedness of the domain. In addition, since we cannot manage directly the energy functional associated with (1) because it might be not well defined, a necessary reformulation of the original problem in a suitable variational setting is performed by using a nice change of variables involving an auxiliary function, delicate to be managed.