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The Maximal Operator, the Variable $L^{p(\cdot)}$, and the Bizarre Geometry of Spaces of Homogeneous Type

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Abstract: A space of homogeneous type (X, d, μ) is a quasi-metric space (X, d) endowed with a doubling Borel measure μ. When comparing quasi-metric spaces to the classical metric spaces, one may perfectly use the ironical saying that "the former are just like the latter except when they are not." Even though their definitions differ only in the triangle inequality, somewhat weakened in the case of a quasi-metric, this little modification leads to unexpected topological properties, such as: in quasi-metric spaces, balls are not necessarily open sets. So first, we will discuss some peculiarities of spaces of homogeneous type as a setting for analysis.

Then a necessary condition for boundedness of the Hardy-Littlewood maximal operator M on variable Lebesgue spaces $L^{p(\cdot)}(X, d, \mu)$ will be presented and proved—however, with the additional requirement on the measure μ also to be *reverse* doubling. Although addressing the special case, this necessary condition is new and still one of a kind. In the Euclidean setting, it has long been known that if M is bounded on $L^{p(\cdot)}(\mathbb{R}^n)$, then necessarily $p_-(\mathbb{R}^n) > 1$, where

$$p_-(X) := \operatorname*{ess\,inf}_{x \in X} p(x)$$

is the essential lower bound of the exponent function $p(\cdot)$. Surprisingly, nothing like this is known in literature for spaces of homogeneous type. We prove that in the setting of the reverse doubling spaces (X, d, μ), the maximal operator is not bounded on $L^{p(\cdot)}(X, d, \mu)$ when $p_-(X) = 1$. This is a joint work with Dr. Oleksiy Karlovych.