MAFIA - the seminar you can't refuse

On the Efimov Effect

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Abstract: For sufficiently short-range attractive potentials it is well known that the one-particle Schrödinger operator has only finitely many bound states (see, e.g., classical results of Zhislin and related criteria). In contrast, this finiteness may fail for three-body systems: Efimov discovered that three particles in three spatial dimensions, interacting via short-range forces, can support infinitely many bound states accumulating at the bottom of the essential spectrum. Predicted in 1970, the effect was proved rigorously by Yafaev (1974), and later given a fully variational proof by Tamura (1991).

A key mechanism behind the Efimov effect is quantum tunneling between threshold configurations associated with *resonant* (zero-energy) states of subsystems. The nature of such threshold states depends strongly on the spatial dimension: for $d \leq 4$ these states are not square-integrable, while still having square-integrable gradient, in sharp contrast to the behavior for $d > 4$.

It was predicted in physics literature (by Nishida) that an Efimov type effect may occur for four particles in two dimensions provided the interaction is purely three-body and each three-body subsystem exhibits a zero-energy resonance. We give the proof for that effect. The relevant underlying dimension in this case is $d = 4$, linking the problem to our earlier analysis of tunneling at the level of resonances in dimension $d = 4$.