MAFIA - the seminar you can't refuse

On the geometry of the p-Laplacian operator

Bernd Kawohl

Universität zu Köln

December 5, 2017
13:15–14:15
in T115/C

Fakultа jaderná a fyzikálně inženýrská ČVUT
Trojanova 13, 12000 Praha

ABSTRACT. The p–Laplacian operator $\Delta_p u = \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right)$ is not uniformly elliptic for any $p \in (1,2) \cup (2,\infty)$ and degenerates even more when $p \to \infty$ or $p \to 1$. In those two cases the Dirichlet and eigenvalue problems associated with the p-Laplacian lead to intriguing geometric questions, because their limits for $p \to \infty$ or $p \to 1$ can be characterized by the geometry of Ω. In this little survey we recall some well-known results on eigenfunctions of the classical 2-Laplacian and elaborate on their extensions to general $p \in [1,\infty]$. We report also on results concerning the normalized or game-theoretic p–Laplacian

$$\Delta_p^N u := \tfrac{1}{p}|\nabla u|^{2-p}\Delta_p u = \tfrac{1}{p}\Delta_1^N u + \tfrac{p-1}{p}\Delta_\infty^N u$$

and its parabolic counterpart $u_t - \Delta_p^N u = 0$. These equations are homogeneous of degree 1 and Δ_p^N is uniformly elliptic for any $p \in (1,\infty)$. In this respect it is more benign than the p-Laplacian, but it is not of divergence type.