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The first Robin eigenvalue for the domains in $\mathbb{R}^n$

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Abstract: The estimates of a k-th eigenvalue $\lambda = \lambda_k(\Omega)$ of the Laplace operator $-\Delta u = \lambda u$ in bounded domain $\Omega \subset \mathbb{R}^n$ are very interesting and long-time question. One of the major book, which laid the groundwork for Spectral Geometry was "The theory of sound" (J. W. S. Rayleigh (1877)). At first, the study of the Dirichlet boundary condition and the Neumann boundary condition was on the front burner. The most famous result in exploration of the extremal eigenvalue is Rayleigh-Faber-Krahn inequality. This states that the first Dirichlet eigenvalue of the domain is always greater than or equal to the first Dirichlet eigenvalue for the ball with the same volume (G.Faber (1923), E.Krahn (1924)). And the same result was giving for the first Neumann eigenvalue, but the ball is a maximiser (G.Szegö (1954), H.F.Weinberger (1956)). As to the Robin boundary condition $\frac{\partial u}{\partial \nu} + \alpha u = 0$, where ν is an outer unit normal vector, α is a real constant, there are not a lot of work with the generalized results. A great interest is an estimate of $\lambda_k^\alpha(\Omega)$ by $\lambda_k^\alpha(B)$, where B is a ball with the same volume of the boundary as the boundary of Ω has (i.e. $|Vol_{n-1}(\partial B)| = |Vol_{n-1}(\partial\Omega)|$).

We are going to discuss the hypothesis

$$\lambda_1^\alpha(\Omega) \le \lambda_1^\alpha(B)\,, \quad \text{if} \quad \alpha \le 0 \quad \text{and} \quad |Vol_{n-1}(\partial B)| = |Vol_{n-1}(\partial\Omega)|\,.$$

We will prove that the hypothesis is true in $\mathbb{R}^3$ in case, when the boundary is diffeomorphic to the sphere and for the domain in $\mathbb{R}^n$ for an arbitrary n with some restrictions on the curvature.